Performance Analysis of Fractional order Multi-Modulus Algorithms for Channel Equalization

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Abstract

In this research work we comparing the performance with the family of blind equalization. These algorithms are CMA, Godard and FrCMA using adaptive filters. We are varying the Signal to Noise Ratio (SNR), convergence factor (step size) µ relatively in different algorithms. The MATLAB tool results exhibits that different algorithms perform different with the same transmitted bits along with noisy environment all algorithms. In blind algorithms no pilot transmitted signal required. There are some difficulties also occur like complexity of the system and cost function. Similarly, the benefit like reduction of the SNR, Intersymbol interference(ISI) and fast convergence also achieved in simulation results. The fractional order MMA algorithms generate improved results which is used in 5G networks, code is general and inevitable need of the current era. The Fractional order Constant Modulus algorithms give improved results like fast convergence as well as steady state condition. FrCMA algorithms is a new approach for practical, general application of the data and wireless communication.

Keywords

FrCMA, Godard, CMA, SNR, ISI, cost function, Fractional derivation methodology

1. Introduction

Equalization is the fundamental part of the cable base or wireless communication. The conventional Blind equalization algorithm used to Minimize Mean Square Error (MMSE) and detecting of the transmitted signal with the absence of additive and multiplicative noise. In this method Constant Modulus algorithm with a special case of the Multi Modulus Algorithm mostly used. [1] Similarly, channel equalization operates without transmitting reference signal using blind equalization algorithms. It bases on the probabilistic and stochastics properties of the transmitting signal. In this paper we discuss Blind adaptive filtering algorithms like Godard, CMA, MMA, FrCM Algorithms. [2]Moreover, we use three-taps impulse using flat frequency response. The objective function of the analysis must be minimization of MSE and combat of the additive White Gaussian noise (AWGN), Intersymbol Interference (ISI), fading. [3] ISI is a multiplicative noise, distortion in the transmitted signal due to multi path loss (Echo). By varying Signal to Noise Ratio (SNR) and convergence factor we improve the fast convergence using algorithms. The comparative simulation results of the algorithms depict in the graphs. [4] We take different taps for frequency selective and flat channels with varying convergence factor, SNR using 4 Quadrature Amplitude Modulation (4QAM) scheme. [5]The channel impairments in digital communication reduce the power of the transmitted signal due to line fading, Intersymbol Interference (ISI), Gaussian noise. [6] Actually multi transmission produce fading which lead the ISI in data communication. Further some duplicate copies of the original transmitted signal occur in output sides show in figure 2.[7] In linear channel produce ISI which a problem for the receiver to receive the transmitted signal. Therefore, blind equalization adaptive filter detects the ISI channel impairment in digital communication. [8] The ISI is a multiplicative noise in wireless communication system. There is a delay version of the transmitted signal received in the output side, which is lead to noise and mitigate the power of the required signal in the output side [10]. The figure1 denotes the ISI impairment in detail. For the purpose to reduce the chance of making an error the receiver will often employ an equalizer in order to reduce the effects of channel distortion. The dispersion causes a distortion of the pulse shape, there neighboring pulses to interfere with each other, resulting in an effect known as inter-symbol interference. [11] We transmit 200 bit in a channel and results take after 5000 iteration using Godard, CMA, FrCMA algorithms. The modulation scheme uses 4 Quadrature Amplitude Modulation (4QAM) and Phase shift keying for all algorithms. Therefore, using Reimann-Livolle (RL) derivation for the cost function parameters values. [12] The final results demonstrate that the fractional order CMA techniques generate minimum MSE, SNR and ISI. It concludes that simulation results show fast convergence and fast steady state condition. [13]

The basic channel equalization block diagram shown in figure 1.In this figure input transmitted signal denoted by $x(k)$, channel impulse response by $h(k)$, Additive white Gaussian noise $n(k)$,$y(k)$ use for output signal, $e(k)$ for error signal, $d(k)$ for desired signal. Similarly, $e(k)= d(k) - y(k)$. Whenever $e(k)$

equal to desire signal d(k) it will be considered ideal case which is free from multiplicative and additive noise. In assume parameters k denote for iteration number [14].

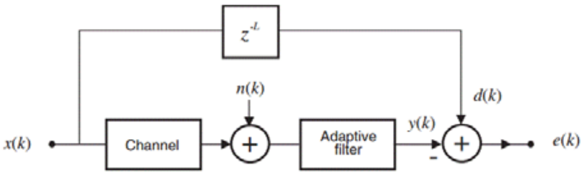

Figure 1. Channel equalization Block Diagram

The remaining section of the paper consist of Literature review, Problem statement, channel equalization, mathematical derivation in algorithms, Simulation results, conclusion and references.

2. Literature Review

The algorithms like Least Mean Square (LMS), Normalized Least Mean Square (NLMS), Recursive Least Square (RLS) are traditional base algorithms. These algorithms are based on the reference transmitted signal. Therefore, the training signal is compulsory in the above mention algorithms. Now we discuss the concept of blind equalization where no reference signal is required. The probabilistic and statistical properties of the transmitted signal use to recover the transmitted signal with the absence of the training signal. [14]

In blind equalization techniques no reference or training signal required in digital communication. Actually, algorithms like CMA, Godard, Sato and FrCMA use the existing stochastic properties of the transmitted signal utilized for the detecting the transmitted signal. There are two types of equalizer such as static equalizer and adaptive equalizer. The static equalizer is compromise in efficiency and reliability. Owing to simplicity and less complexity is a positive aspect of the static equalizer. It is very hardship to make an equalizer which is work out without knowing transfer function and impulse response of the flat channel response. In adaptive filter equalizer transfer function of the three impulse response automatically adjusted to the selected algorithms. The utmost flavor of the adaptive filter is furnishing the deficiency of the static equalizer. [15]

3 Model of the Channel equalization and Methodology

In figure 1 Channel equalization block diagram consist of the input signal, Additive white Gaussian Noise (AWGN), multiplicative noise ISI desired signal, error, delay version of the signal, output signal

The input x (k) convolve to the channel impulse response h(k) and add some additive noise n(k). Therefore, output y(k) is a delay version of the input signal.

$$Y(k)=x(k)h(k)+n(k) \quad (1)$$

The output is always equal to the convolution of the input signal and impulse response along with additive noise as shown in equation 1

The impulse response equation along with additive noise as follow

$$Y= hx^T+n \quad (2)$$

Where in equation (2) $(.)^T$ use for transpose

$$h =h(x)+g \quad (3)$$

The input vector matrix

$$x= [x(n)\ x(n-1)\ x(n-2)]^T \quad (4)$$

The impulse response is

$$h=h_0 +h_1 Z^{-1} +h_2 Z^{-2} (5)$$

when we substitution of the equation (4) and equation (5) the new equation as follow

$$Y=h_0 x(n)+h_1 x(n-1) +h_2 x(n-2) + n \quad (7)$$

3.1 Godard Algorithm

The basic objective of the Godard algorithm is to mitigate the cost function as follow

$$|\varepsilon_{Godard}|_{min}=|E[(|w(k)x(k)-r_q)]|_{min} \quad (8)$$

$$\varepsilon_{Godard} =E[e^p_{Godard}(k)] \quad (9)$$

In equation 8 Godard cost function equal to the difference of the convolution y(k) of the weight w(k), and input signal x(k)) and constant value

Where ε_{Godard} objective function of the Godard algorithm, E for expectation, r_q denote level in Godard algorithm, q is a positive integer. Godard algorithm use for high order statistic of the constellation implementation which is an improve mechanism for research. [16]

In Equation 8, E represents mean value of Godard algorithm, w represents Matrix of weighted values, x is the matrix of the input values, while k is used for vector values of initialization time. I used 0,1,2… for initial values of K iteratively.

We can assume y(k) as k=0,1, 2,.....k

$$y(k)=w^H(k)x(k) \quad (10)$$

In equation 10 w denote weight, x(k) input signal, $(.)^T$ use for transpose, y(k) received signal

$$\varepsilon_{Godard}=E[(|y(k)|^q-r_q)^p] \quad (11)$$

In equation 11 p, q are positive integers in Godard algorithm, r_q is constant value for constellation diagram, ε_{Godard} denotes Godard cost function

When we put value q and p both equal to 2 in equation (11)

$$\varepsilon_{Godar}=E[(y(k)|^2-r_q)^2 \quad (12)$$

In equation 12 square open of the cost function and put q = 2 as follow

$$\varepsilon_{Godard=}E[(y(k)|^4-2E[|y(k)|^2r_2+r_2^2] \quad (13)$$

In equation 12 E is use for required expected mean values

Initialization of the Godard algorithm assume for the experimental research. The random

The stochastic gradient equation of the Godard algorithm as follow taking derivative with respect to weight of Godard cost function equation 2.7 using chain rule

$$\left(\frac{d}{dx}(uv) = u\acute{v} + v\acute{u}\right)$$

$$\frac{d}{dw}(|y(k)|^{q}-r_q)^{p} = p\,|(y(x)|^{q}-\gamma_q)|^{p-1}\frac{d}{dw}(|y(k)|^{q})$$

$$= p\,|(y(x)|^{q}-\gamma_q)|^{p-1}q(|y(k)|^{q-1})\frac{d}{dw}(w^{H}(k)x(k)-\gamma_q)$$

put $y(k)= w^{H}(k)x(k)$

$$=pq(|y(x)|^{q}-\gamma_q)|^{p-1}|y(k)|^{q-1})x(k)$$

$$\frac{d}{dw}(|y(k)|^{q}-r_q)^{p}=pq(|y(x)|^{q}-\gamma_q)|^{p-1}|y(k)|^{q-2})\,y^{*}(k)x(k) \quad (14)$$

The modified Godard equation after differentiating the objective function equation 14

$$w(k+1)=w(k)-\frac{1}{2}\mu pq|y(k)|^{q}-\gamma_q)^{p-1}|y(k)^{q-2}y^{*}(k)x(k) \quad (15)$$

$$w(k)=w(k-1)-\frac{1}{2}\mu pq|y(k)|^{q}-\gamma_q)^{p-1}|y(k)^{q-2}y^{*}(k)x(k) \quad (16)$$

The equation 16 substitute with the equation 12

$$w(k)=w(k-1)-\frac{1}{2}\mu pq e^{p-1}_{Godard}(k)|y(k)|^{q-2}y^{*}(k)\,x(k) \quad (17)$$

put p=2 , q=1 in equation 16 updated form of Godard (CMA) we get

$$w(k+1)=w(k)-\mu|y(k)|^{-1}-\gamma_1)^{-1}|y(k)^{-1}y^{*}(k)x(k) \quad (18)$$

Now put p=2 and q=2 in equation 15 we get equivalent equation of the Godard (CMA) algorithm

$$w(k+1)=w(k)-2\mu|y(k)|^{2}-\gamma_2)^{1}|y^{*}(k)x(k) \quad (19)$$

3.2 Constant Modulus Algorithm (CMA)

In blind adaptive equalization method CMA algorithm reduce the distance between equalizer output and some constant. CMA algorithms also a training less algorithm like MMA and special family of blind adaptive equalization algorithms. In objective function equation of CMA algorithm s(t) used for transmitting signal, J for channel delay time, C denote constellation set, h for impulse response

$$x(k+J)=s(k)h(J)+\left(\sum_{l=\infty}^{k+j} s(l)h(k+J-l)\right)+n(k+J) \quad (20)$$

The cost function of the Sato algorithm as follow

$$e_{CMA}(k) = E[(y(k)|^{2}-r_q)$$

where e_{CMA} constant modulus algorithm cost function equal to $y(k)|^{2}-\gamma_2|$ the new CMA algorithm equation as follow

$$w(k+1)=w(k)-2\mu e_{CMA}y^{*}(k)x(k) \quad (21)$$

In CMA algorithm where $y(k)= w^{H}(k)x(k)$, y(k) use for output signal, x(k) for input signal and $k \geq 0$ and x(0)= w(0) represent random vectors in equations.

3.3 Multi- Modulus Algorithm (MMA)

Multi-Modulus algorithm is a traditional blind equalization algorithm. This algorithm has some deficiency like slow convergence in high order modulation channel like 4QAM,16QAM, 64QAM.Moreover MMA algorithm ISI rate is greater as compared to proposed Fractional order CMA algorithm. For reduction of steady state mis adjustment as well as bit error rate we use a modified algorithm Godard (CMA) , FrCMA varying the values of parameters q and p. The tab weights in next chapters control the intensity of noise in the output side of the equalizer. The tab weights are frequently reduced according to the output (cost) function. Similarly, the output function of the MMA written in form of real (R) and imaginary (I) in equation form as follow

$$€MMA=€R(n)+€I(n) \quad (21)$$

In equation 21 € denote cost function, and R , I use for real and imaginary values simultaneously

Thus

$$€MMA=E\{[|y_r(n)-c_q)|\}+E\{[y_i(n)|^{2}-c_q]^{2}\} \quad (22)$$

Equation 22, $c_q$denote constellation level in blind equalization CMA algorithms, E denote mean value, y(n) output signal

The weighted tab equation of the MMA algorithm as follow

$$w(k+1)=w(k)-\mu.e(n)y(n) \quad (23)$$

In equation 23 $e(n) = e_r(n) + ie_i(n)$

In Equation 23 µ use for convergence factor , e(n) error signal, and y(n) for output signal

3.4 FRACTIONAL ORDER CMA

The output signal denoted by y(k), channel finite impulse response assume $h=[h(k),h(k-1),\ldots\ldots x(k-N+1)]^{T}$ where $(.)^{T}$ express transpose k assume symbol, and k-1 assume previous symbol. These symbols in transmission channel also cause of Intersymbol Interference which is multiplicative noise. For the elimination of AWGN noise and ISI I will design a blind equalizer of N weights w=[w(k), w(k-1),……..w(k-S+1)]^T for the purpose of mitigation of the Mean Square Error(MSE) using fractional order Godard algorithm cost function. Assume parameter € denote cost function as

$$€= E[(|y(n)|^2 - c_q)^p] \qquad (24)$$

In equation 24 c_q use for constellation level constant, p and q are parameters of fractional order CMA algorithm, where y(k)= $w^T(k)x(k)$, $(.)^T$ define Hermitian transpose

$$c_q = \frac{E[|s(k)|^{2q}]}{E[|s(k)|^q]} \quad (25)$$

Proposed Fractional order CMA algorithm is the combination of the conventional Godard and fractional Godard CMA with varying parameters with case 1 and case 2 p,q (2,2) , p,q (2,1)

$$w(k)= w_g(k) + w_{f(k)} \qquad (26)$$

In equation (26) $w_g(k)$ denote Godard algorithm weights, $w_{f(k)}$ use for fractional algorithm weights. The Godard (CMA) algorithms after derivation with respect to w of the cost function incorporating equation (11) and (14) can be simplified is as follow: assume $\gamma_{q} = c_q$

$$w_g(k)=w(k-1)-\frac{1}{2}\mu pq(|y(x)|^q - c_q)^{|p-1}|y(k)|^{q-2})y^*(k)x(k)$$
(27)

In equation 27 $(.)^*$ express complex conjugate, μ for step index, p and q are Godard (CMA) algorithm varying parameters, y(x) show output signal, x(n) input signal. Now we use the fractional order CMA algorithms to reduce the ISI and improve convergence rate. Therefore, we simplify the Reimann-Livolle (RL) equation

$$_aD^p_t(t-a)^u=\frac{\Gamma(u+1)}{\Gamma(u-p+1)}(t-a)^{u-p} \qquad (28)$$

In equation 29 (p< 0, u> -1), a and t are lower and upper limit, p and u are varying parameters assume in RL derivative equation. Putting € instead of cost function in place of t-a is as follow

$$_aD^p_t(€)^u=\frac{\Gamma(u+1)}{\Gamma(u-p+1)}(€)^{u-p} \qquad (29)$$

Assume p=u in equation 28 the new equation is

$$_aD^u_t(€)^p=\frac{\Gamma(p+1)}{\Gamma(p-u+1)}(€)^{p-u} \qquad (30)$$

Put the values of p= 1 in equation (30)

$$_aD^u_t(€)=\frac{\Gamma(1+1)}{\Gamma(1-u+1)}(€)^{1-u} \qquad (31)$$

$$_aD^u_t(€)=\frac{\Gamma(2)}{\Gamma(2-u)}(€)^{1-u} \qquad (32)$$

and assume u> -1 for the convergence of the integral. Similarly, the function f(t) and fractional derivative combine equation is discussed

$$_aD^u{}_{,x}f(t) = \frac{1}{\Gamma(k-p)}\left(\frac{d}{dt}\right)^k\int_a^t(t-\tau)^{k-p-1}f(\tau)d(\tau) \qquad (33)$$

where a and t is the limit of integration, f(t) = $(€)^p$ assume equal to the cost function of Godard fractional order algorithm in equation (11) In equation (33) Γ (k-p-1)=$\int_0^\infty e^{-t}\,t^{z-u}$dt

and € is greater than o, RL based fractional order derivative of fractional order u> -1

The function is as follow [8], the modified fraction equation of equation (33) when f(t) = f(x)

$$_aD^u{}_{,x}f(x)={}_cD^u{}_{,x}f(x)=\frac{d^m}{dx^m}\left[\frac{1}{\Gamma(p)}\int_0^x(x-t)^{p-t}f(t)dt\right] \qquad (35)$$

The simplified form of the equation (35) is as follows;

$$_aD^u{}_{,x}f(x)=\left[\frac{1}{\Gamma(1-u)}\int_0^x\frac{f(x)-f(t)}{u}f(t)\,d(t) \qquad (36)$$

In equation (36) :. x > 0, and n-1< u ≤ n

Where u is fractional order and n is an integer. Therefore, we use Reimann-Livolle fractional equation;

$$_aD^u{}_{,t}€^p=\frac{p+1}{(p-u+1)}€^{u-v} \qquad (37)$$

In equation (37) € express cost function of the fractional order CMA algorithm. We put p =1 the modified equation as;

$$_aD^u{}_t€^p=\frac{2}{(2-u)}€^{u-v} \qquad (38)$$

The fraction equation is;

$$w_f(k)= -\mu_f D^u(|w^H(k)x(k)| \qquad (39)$$

The derived form of fraction and conventional algorithm are; $_aD^u{}_{,t}€^p$ where cost function integral form is;

$$€=\int_0^\infty e^{-t}\,t^{z-u}\,dt$$

$$€ \quad = -t^{z-u}(e^{-\infty} - e^{-0}) = t^{z-u} \qquad (40)$$

In equation (36), f (t) assume $€(t)^z$. In equation (40), fractional constant u must be greater than -1. Now applying fractional derivation in equation (36) with respect to weight;

$$ {}_0D^{u}{}_{,t}\, €^{z} = \frac{z+1}{(z-u+1)} €^{z-u} \qquad (41)$$

Putting z = 1 in equation (41)

$$ {}_0D^{u}{}_{,t}\, €^{z} = \frac{2}{(2-u)} €^{1-u} \qquad (42)$$

The cost function $€(t)^p = f(t)$ the RL fraction equation equals ${}_0D^{u}{}_{,t}\, f(t) = {}_0D^{u}{}_{,t}\, f(x)$

$$ {}_0D^{u}{}_{,t}\, f(t) = \frac{1d^n}{\Gamma(n-u)dx^n} \int_a^x (x-\tau)^{n-u-1} f(t)\, d(\tau) \qquad (43)$$

For fractional using equation (40) assume z = p = 1

$$ {}_0D^{u}{}_{,t}\, €^{z} = \frac{z+1}{z-u+1}\; €^{z-u} \qquad (44)$$

The simplified form of equation (44) is as follows;

$$ {}_0D^{u}{}_{,t}\, €^{z} = \frac{2}{2-u}\; €^{1-u} \qquad (45)$$

Now weight of the fractional equation and cost equation derivative with respect to weight are represented as;

$$ w_f(k) = w\,(k-1) - \mu pq(|y(k)|^q - c_q)^{p-1}|y(k)|^{q-2}\, y^*(k)x(k) \qquad (46)$$

Equation (46) $w_f(k)$ equals to $-\mu\; {}_0D^u\; y(k)$. The simplest form of equation (46) is as follows;

$$ w_f(k) = -\mu pq\; {}_0D^u\; |y(k)|^{q-2}\, y^*(k)x(k) \qquad (47)$$

Now we multiply equation (46) fraction simplified equation with equation (47)

$$ w_f(k) = -\mu pq\,(|y(k)|^q - c_q)^{p-1}|y(k)|^{q-2}\, y© (k)x(k)\, {}_0D^u \qquad (48)$$

Thus incorporating equations (45) and (48)

$$ w_f(k) = -\mu pq\,(|y(k)|^q - c_q)^{p-1}|y(k)|^{q-2}\, y© (k)x(k) * \frac{2}{2-u}\; €^{1-u} \quad (49)$$

In equation (49) (©) is used for the multiplication of the fraction part. Further $€^{1-u} = w_G^{1-u}$ (k) is used for cost weight function of the Godard algorithm

$$ \boldsymbol{w}_f(k) = -\mu pq\,(|y(k)|^q - c_q)^{p-1}|y(k)|^{q-2}\, y^*(k)\mathbf{x}(k)©\frac{2}{2-u}\; w_f^{1-u} \quad (50)$$

We know that output equation equals $y(k) = w^H(k)\, x(k)$. Thus putting the algorithm parameters values of p and q in equation (50) i.e. p=2,q=1 for Godard CMA algorithm and p=q=2 for MMA algorithm, the derived equation (27) comes to (by using value convergence factor μ=0.5);

$$ \mathbf{w}_G(k) = -0.5pq(|y(x)|^q - c_q)|^{p-1}\, |y(k)|^{q-2})\; y^*(k)\;\mathbf{x}(k) \qquad (51)$$

For fractional CMA put p=2, q=1 in equation (51) then the modified form along with RL fractional derivation is as follows;

$$ \boldsymbol{w}_f(k) = -\mu\,(|y(k)| - c_1)|y(k)|^{-1}\, y^*(k)\mathbf{x}(k)©\frac{w_f^{1-u}}{2-u}(k) \qquad (52)$$

Now the fractional simplified MMA equation using the parameters p = q = 2 is updated as equation (52);

$$ \boldsymbol{w}_f(k) = -2\mu\,(|y(k)|^2 - c_2)|y(k)|y^*(k)\mathbf{x}(k)©\frac{w_f^{1-u}}{2-u}(k) \qquad (53)$$

Now we take Godard fraction derivative with respect to weight by putting in value of parameters p = 2, q = 2 on the base of equation (49)

$$ \boldsymbol{w}_G(k) = -8\mu\,(|y(k)|^2 - c_2)|y(k)|^0\, y^*(k)\mathbf{x}(k)©\frac{w_f^{1-u}}{2-u}(k) \qquad (54)$$

Now the Godard and fraction weight equation using equation (53) and (54) $w(k) = \boldsymbol{w}_G(k) + \boldsymbol{w}_f(k)$

$$ w\,(k) = -10\mu\,(|y(k)|^2 - c_2)y^*(k)\mathbf{x}(k)©\frac{2}{2-u}\; w_G^{1-u} \qquad (55)$$

Now for simulation results constellation level constant is $c_q = c_1 = \sqrt{2}$ for q=1 and c_2=2 for q=2 .Therefore we assume parameters p=1 , q = {1, 2} for QPSK modulation scheme using cost function of fundamental MSE equation

$$ MSE = E\,[\,(|y(k)|^2 - c_2)^2] \qquad (56)$$

The Godard MMA stochastic equation comes to;

$$ w\,(k) = w(k-1) + \mu_1 €_{CMA}\, y^*(k)\mathbf{x}(k) + \mu_2 €_{MMA}\, y^*(k)\mathbf{x}(k)\; \frac{w(k)^{1-u}}{\Gamma(2-u)} \qquad (57)$$

In equation (48), μ_1, μ_2 are step size parameters of the algorithm. Fraction CMA (2,1) and frMMA (2,2) can be summarized accordingly. The conventional CMA Godard uses the final update of the frCMA. In simulation, conventional CMA uses the fractional CMA iteration for modified results. The final weights is the sum of the Godard CMA and frCMA.

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It is also noted that increasing fraction order (u) from 0 to 1, the convergence also enhances. In next chapter we will implement these algorithm parameters for improved simulation results [2,8,9].

4 Simulation Results

In simulation results part we show comparative results of the same parameters of Godard CMA and FrCMA algorithms. I have tested two algorithms Godard CMA (2,2) and Fractional CMA (2,1) to test impact for noise cancellation using blind equalization. For implementation I used MATLAB. I have used 4 Quadrature Amplitude Modulation (4QAM) for randomly generated data bits transmission. I have molded of 3 tap case 1 for flat fading response of impulse channel like h=[0.7 + j0.2, 0.1-j0.02, -0.01-j0.001]. The quantity of equalizer used 3 tap along with delay 0. We took 400 symbols and simulate the results after 5000 iteration. The 200 symbols are transmitted for two algorithms. I took the convergence factor (step size) between 0 and 1 (0< 1) . Signal to Noise Ratio (SNR) select 20, 25 dB for channel 1 flat fading channel. The values of parameters p and q =2 for channel1. The delay for chnnel1 is 0. The results analytically compare with respect to fast convergence and steady sate condition with proposed fractional order CMA algorithm.

Similarly, we tested channel 2 as a frequency selective channel using 14-tab equalizer. The impulse response of channel 2 (frequency selective channel) are h= [0.2231 -j*0.1745, -0.0077 +j*0.00281,0.3312+j*0.4829, 0.1703 +j*0.0282,-0.1024+j*0.1293,0.0743-j*0.0580,0.0070-j*0.0642, 0.0340-j*0.0442,-0.0191+j*0.0023, 0.0060-j*0.0076, 0.0035+j*0.0133, -0.0015-j*0.0067,0.0092-j*0.0045, -0.0022-j*0.0003]; The frCMA algorithms parameters are fractional order μ, fractional step index μ_f , step index u range between 1 and 0. In channel 2 a fix SNR 30dB. The parameters p=2, and q=1 fix in channel2 with varying other parameters like fractional order, step index, fractional step index.

4.1 Learning performance curves

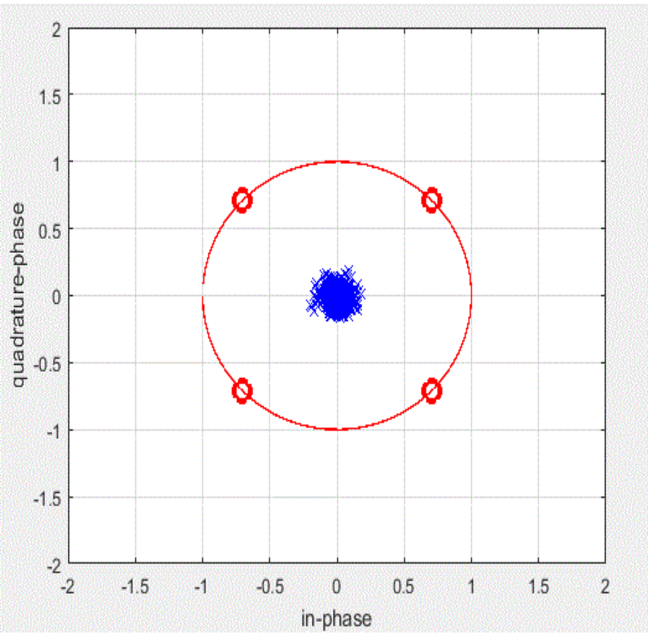

Figure 2: Equalized signals for CMA, FrCMA algorithm using the first coefficient initialization.

The output learning curve of the two algorithms disclose the convergence in two-dimensional graph. The horizontal line show x-axis for iterations and vertical line denote the Mean Square Error in range of 10^{-2} to 10^{0}.The receiver signals before equalization the horizontal line show in-phase in range of -2 to 2. In the vertical line show the quadrature-phase in range of -2 to 2.

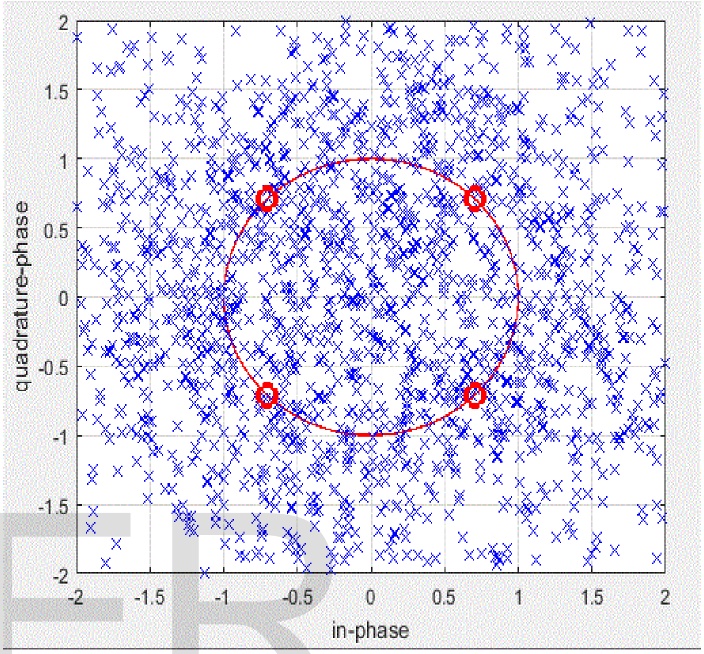

Figure: 3 Equalized signals for CMA, FrCMA algorithm using the Second coefficient initialization

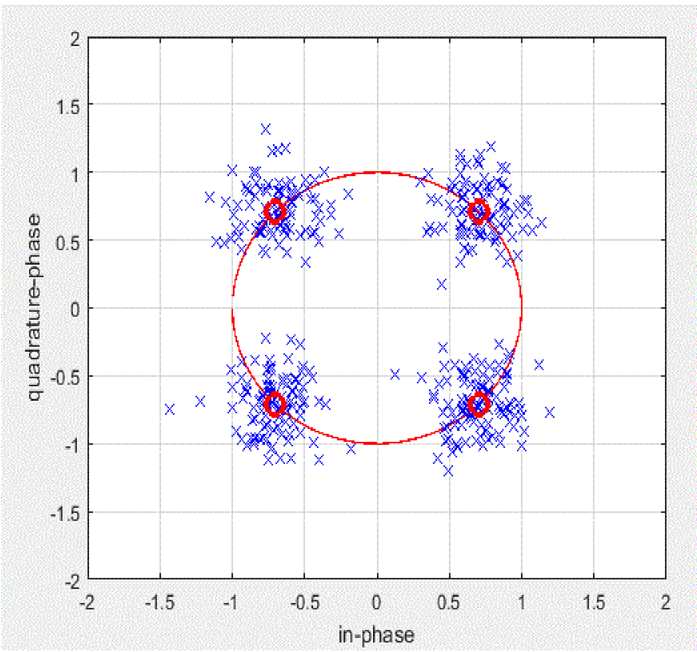

Figure 4: Equalized signals for the Weiner filter SNR 25dB

Table 1: Comparative results of two algorithms on fix SNR 30dB in figure (5)

S/N	**Algorithm**	**Iteration required for convergence**	**Minimum MSE**
1	Standard CMA	335	0.2122
2	Fractional CMA	183	0.1145
3	Standard CMA	106	4.087×10^{-1}
4	Fractional CMA	10	8.81×10^{-2}

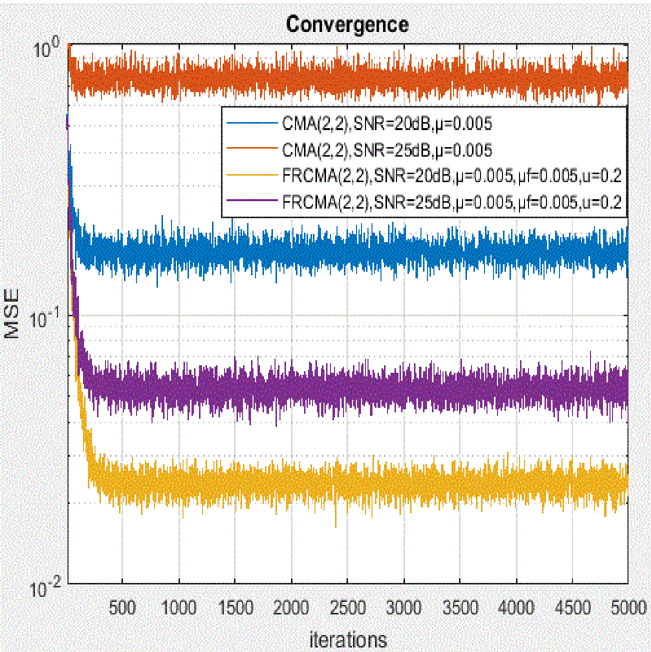

Figure 5: Learning curve for the CMA and FrCMA algorithm

We generated a 200 bits random data along with uniform distribution and transmitted signal is distributed by the channel impairment, AWGN, ISI, fading and phase distortion. The Modulation scheme we used 4QAM samples for unitary power. Experimental initialization of standard CMA (Godard) and fractional order CMA algorithms.

Impulse response h = $[0.7 + j*0.2, 0.1-j*0.02, -0.01-j*0.001]^T$

And additive white Gaussian noise with variance $10^{-2.5}$, mean μ =0, SNR =2 0dB and 25dB ,convergence factor 0.005 we run the CMA(2,2) , Convergence factor assume μ =0.2 , Now for FrCMA algorithm Convergence factor assume 0.005 along with SNR =20dB ,u=0.2 , similarly the positive integer parameters of the CMA(2,2) and FrCMA(2,2) algorithm assume p = 2 and q = 2,fractional variant u=0.005 SNR =25dB these algorithms are operate for 5000 time to achieve extreme correct simulation results for research.

The input signals of the two algorithms CMA (2,2) Algorithm, FrCMA are the same. Figures 3, depicts first coefficient initialization in vertical line quadrature phase between -2 to +2 and horizontal line in phase also between -2 to +2.

The figure 4 shows the second coefficient initialization in vertical line quadrature phase between -2 to +2, horizontal line in- phase between -2 to +2. The figure 4.3 represents the equalized signal of the wiener filter also exhibits in vertical line quadrature phase between +2 to -2 and horizontal line between -2 to +2. We explore the input signals all SNR 20dB,25dB.

Figure 6 exhibits the learning curve of the two algorithms after 5000 iteration K in horizontal line and vertical line show Mean Square Error between 10^{-2} and 10^{0}.

Results of MSE produce linear in Figure 6 by applying varying SNR 20dB and 25dB. Similarly using fix convergence factor 0.2 and fractional convergence factor 0.005, fractional order 0.2 fix. When transmitting power is kept 1 and noise power is taken 10^(-1) for SNR 20dB .Moreover SNR 25 dB assume transmitting power 100 and noise power 10^(-0.5)

It concludes that 20dB FrCMA results improved as compare to the results of the CMA algorithm using same parameters exhibits fast convergence figure 6.

In channel 2 we take fourteen-tab and test using 200 bits randomly. There are a uniform distribution and transmitted signal is distributed by the channel impairment, AWGN, ISI, fading and phase distortion. In fourteen-tab complex frequency selective channel2 we sustain SNR 30dB for standard CMA and FrCMA algorithm simultaneously. The Modulation scheme we used 4QAM samples for unitary power. Experimental initialization of standard CMA (Godard) and FRCMA algorithms.

Impulse response h = [0.2231 -j*0.1745, -0.0077 +j*0.00281,0.3312+j*0.4829, 0.1703 +j*0.0282, -0.1024+j*0.1293,0.0743-j*0.0580,0.0070-j*0.0642, 0.0340-j*0.0442, -0.0191+j*0.0023, 0.0060-j*0.0076, 0.0035+j*0.0133, -0.0015-j*0.0067,0.0092-j*0.0045, -0.0022-j*0.0003];^{T}

And additive white Gaussian noise with variance $10^{-2.5}$, mean μ =0, SNR =30dB fix for CMA and FRCMA algorithm, convergence factor 0.379, fractional convergence factor 0.379 and fractional order 0.9. We run the CMA(2,2) , Convergence factor assume μ =0.02 , Now for FrCMA algorithm Convergence factor assume 0.002 and 0.01 along with fix SNR =30dB ,u=0.2 , similarly the positive integer parameters of the CMA(2,2) and FrCMA(2,2) algorithm assume p = 2 and q = 1,fractional variant u=0.02 and 0.9 these algorithms are operate for 5000 time to achieve extreme correct and improved simulation results for research .

Figure 10 exhibits the learning curve of the two algorithms CMA and frCMA after 5000 iteration in horizontal line and vertical line show Mean Square Error (MSE) between 10^{0} and 10^{-2}.

Results of MSE produce different in two algorithms in Figure 10 by keeping fix SNR and varying convergence factor, fractional order, fractional convergence factor, delay and generate convergence in case 2. When transmitting power is kept 10 and noise power is taken 10^{-2}. Further the tuning factor like convergence factor we keep different in algorithms CMA, frCMA algorithms 0.02, 0.379, respectively. Similarly, fractional constant values also different like 0.7 for CMA and 0.9 for frCMA algorithm. The algorithm parameters p and q must same as 2 in both algorithms. Filter order no denoted from N is one in both algorithms for fast convergence.

It concludes that SNR 30dB exhibits fast convergence in frCMA as compare to standard CMA algorithms for convergence show in comparative Log Scale figure 10.

The figure 10 show that when we use SNR 30db using algorithm frCMA minimum MSE 8.8 x 10^{-2} after 10 iterations. In second case Algorithm CMA achieve MSE 1.145 x

10^{-1} and frCMA achieve MSE. Similarly, algorithm frCMA convergence is extremely faster on the same parameters of algorithm. Again, we run both algorithms using different parameters, like convergence factor 0.02 for CMA and 0.01 for frCMA algorithm. Moreover, fractional order for frCMA algorithm kept 0.7 in second execution. The output results again compare with respect to fast convergence, steady state condition and minimum MSE.

Thus, it concludes that using fix SNR 30dB frCMA Algorithm is fast converge and improved MSE on 10 iteration and get minimum MSE 8.8 x 10^{-2} and CMA algorithm taking 335 iteration for convergence achieving MSE 0.1145. Further algorithm CMA is the worst case for fix SNR as well as for different SNR. The figure 10 depicts the fast convergence in after a few iteration and also minimum iteration as compare to other cases of CMA algorithm with same and different parameters.

The performance analysis of the FrCMA and Godard CMA are exhibits in the analytical results of these algorithms with respect to alter signal to noise ratio, fast convergence to achieve improved results. The table 4.2 denote the CMA and frCMA algorithms parameters across flat frequency response channel (channel 1) is as follow. The difference of the MSE for SNR 20dB and 30dB and CMA verses frCMA are shown in table 2.MSE decreases in fractional order CMA algorithm as compare to conventional CMA algorithms with same parameters.

In above figures 6, 10 disclose that when SNR increase MSE decrease. Moreover, SNR inversely proportional to fast convergence in mention algorithms. Similarly, when we increase SNR 30dB and fix other parameters then frCMA algorithm converge after 9 iteration in fraction order 0.9 as exhibits in figure 10. The MSE on frCMA algorithm 8.48 x 10^{-2}. On the other hand, CMA algorithm using same parameters taking 232 iteration for convergence on the same time with MSE 1.227x10^{-1}

Table 2 Observation of MSE of the algorithms with fix 30dB SNR and variable assumed parameters for figure (6)

S/n	algorithm	SNR	μ	μ_f	U	MSE
1	CMA (2,1) Channel 1	30dB	0.0020	0	0	3.977 X 10^{-1}
2	frCMA (2,1) Channel 1	30dB	0.0020	0.0020	0.5	2.136 X 10^{-1}
3	CMA (2,1)	30dB	0.0020	0	0	1.227 X 10^{-1}

	Channel 2					
4	frCMA (2,1) Channel 2	30dB	0.0020	0.0020	0.5	8.48 X 10^{-2}

Table 3: Experimental results of algorithms with fix SNR and variable parameters in figure (7)

SNR	Algorithms	MSE
20dB	Standard CMA (2,2) Channel 1 frCMA (2,2) Channel 1	1.8×10^{-1} 2.415×10^{-2}
20dB	Standard CMA (2,2) Channel 2 frCMA (2,2) Channel 2	8.159×10^{-1} 5.705×10^{-2}

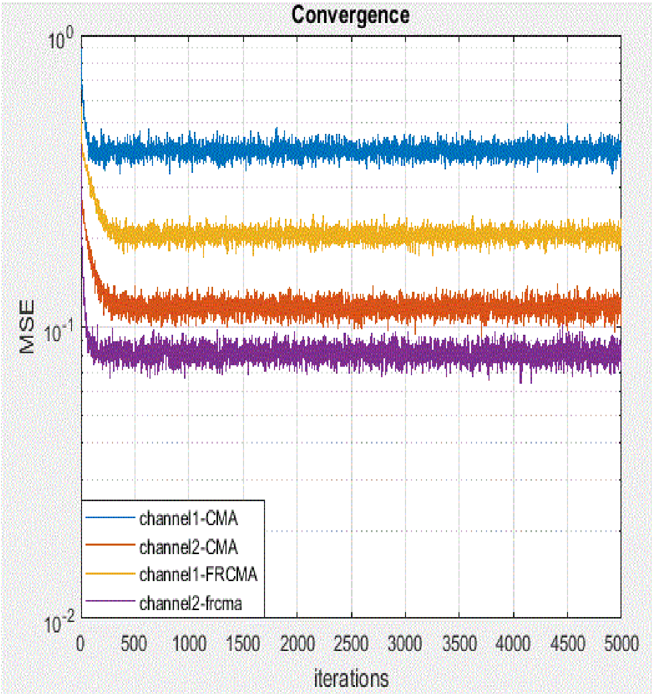

Figure 6: Comparative Learning curve for the CMA, frCMA algorithm

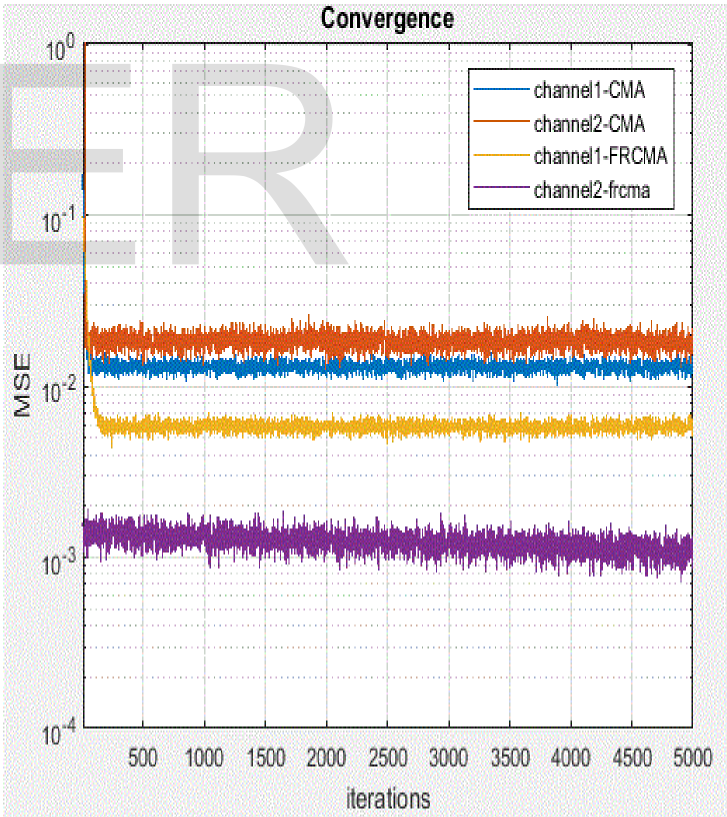

Figure 7: Comparative Learning curve for the CMA, frCMA algorithm

Table 5: Experimental results of algorithms with fix SNR and variable parameters in Figure (8)

Channel	algorithm	Step size	uf	u	SNR	Iteration	MSE
Channel1	CMA	0.05	0	0	20	78	2.18
Channel1	frCMA	0.05	0.05	0.8	20	20	0.181
Channel2	CMA	0.05	0	0	20	268	2.63 x 10^-2
Channel2	frCMA	0.05	0.05	0.8	20	2	2.48 x 10^-4

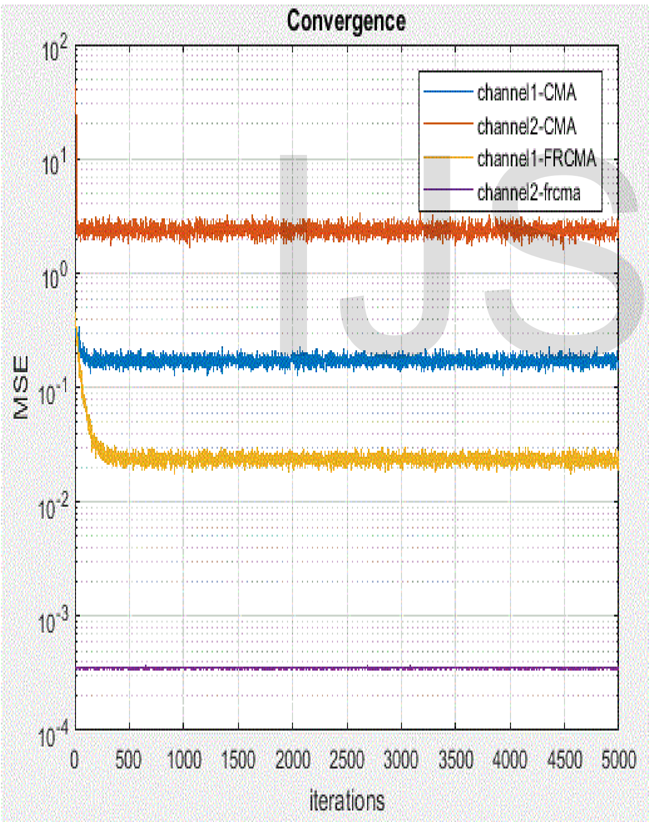

Figure 8: Learning curve for the CMA, frCMA algorithm

5 Conclusion

We precisely conclude that the fractional order Constant Modulus Algorithm results are improved in form of fast convergence. The FrCMA MATLAB results improved as compare to conventional CMA algorithms with increasing SNR as well as reducing ISI.Similarly, a smaller number of iterations required using fractional order Constant Modulus Algorithm.

This research describes analytical comparative study of the Blind equalization algorithms. The core aim is optimization of the signal power and simultaneously combating noise power along with multiplicative ISI. For this purpose, we have used algorithms like standard CMA Algorithm, frCMA with varying SNR while fixing other algorithm parameters. In frequently used algorithms in blind adaptive class family. The purpose of the research to minimize the least square error and mean square error. Similarly, achieving fast convergence and steady state condition are also our aim of the research.

The chief contribution of the research is comparison of the transmitted signals and receiving signals using different algorithms with MATLAB tool results. Similarly, the result taken after a thousand iteration for the scrutiny of research. The final decision taken till to 5.705 X 10^{-2} minimum MSE as compare to all previous results.

Finally, in some aspects on frCMA algorithms results is dominant as compare to other conventional CMA algorithms. These results taking considering minimum MSE, less number of iterations required for the fast convergence and maximum SNR. The analytical derivation of the algorithms using adaptive filter. The fractional order algorithm applicable for next generation, fast steady state convergence which is the need of data communication. Similarly, there are some complexity which is require further research.

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